Disturbance Attenuation in Fault Detection of Gas Turbine Engines: a Discrete Robust Observer Design

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***Abstract*—This study is motivated by the on-board fault detection of Gas Turbine Engines (GTEs) where the computation resources are limited and the disturbance is assumed to be band-limited. A Fast Fourier Transformation (FFT)-based disturbance frequency estimation approach is proposed and performance indices are improved by integrating such frequency information. Furthermore, in the left eigenvector assignment, both eigenvalues and free parameters are optimized. As illustrated in the application to the actuator fault detection of a GTE, significant improvements are achieved compared to the existing methods. By combining the frequency estimation and eigenvalue optimization, the main contribution of the paper is the reduction of the computation complexity and the avoidance of the local optimal solution due to fixed eigenvalues.**

***Index Terms*—Fault detection, fast fourier transformation, gas turbine engine, robust observer design**

I. Introduction

The design of RFDO (Robust Fault Detection Observer) has received much attention in recent years. (see, e.g., [1], [2], [3], [4], [5], [6]). In the optimal observer design that aims at enhancing the robustness to disturbances and the sensitivity to faults, the basic concept is to measure the robustness and sensitivity by a suitable performance index and optimise it. With the aid of well-established robust control theories, a lot of performance indices have been proposed, such as H_2 [3], H_2 in time domain [7], H_∞ [5], H_- [8], [9], and mixed H_-/H_∞ [8].

One of them, eigenvalue assignment [10], [11], [12] shows a lot of advantages when applying to observer designs. As a parametric pole assignment method [11], [13], it assigns the closed-loop poles to desired places arbitrarily [10]. It is well known that the solution is not unique which enables the optimal fault detection observer design. Moreover, through parameterizing the performance index, the eigenvalue assignment based RFDO design turns into an optimization problem [1], [10], [11].

In the application to on-board condition monitoring of GTEs [14], because of the limited computation resources, a fast RFDO design is required. However, the traditional $H_2(H_\infty)$-norm based RFDO design demands relatively more computation due to the fact that an integral or griding over the whole frequency range are required. Moreover, the H_∞ observer is designed to minimize the peaks of transfer functions at some frequency w_p for the worst-case. Note that w_p is determined by the transfer functions (system matrices), not by disturbances. Since it is more likely that the disturbance frequency $w_d \neq w_p$, the H_∞ RFDO that gives the basic guarantee of the performance at the worst-case may be too conservative in some application cases.

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In many industrial applications, the disturbance can be treated as a semi-stochastic process with main contents on some frequency w_d, instead of a Gaussian noise uniformly distributing over the whole frequency range. This disturbance assumption makes sense in a lot of practical applications, such as GTEs. By optimizing the performance indices at w_d, instead of at the worst case (which requires H_∞ optimization over the whole frequency range), the resulting observer should have a better disturbance attenuation performance.

Furthermore, in controller designs [11], [13], the pole positions have been pre-specified according to control performance specifications. In observer designs, however, there is no an explicit way to determine the best positions of poles. Since the positions of eigenvalues affect the observer performance greatly, keeping eigenvalues fixed and optimizing free parameters alone may not give a global optimal solution.

Keeping these two points above in mind and assuming the band-limited disturbance is unknown, we proposed an approach to estimate the disturbance frequency via spectra analysis of residuals. Such frequency information is then integrated to form an improved frequency-dependent performance index for reducing the computation costs and enhancing disturbance attenuation. In the optimization procedure, both pole positions and free parameters are optimized simultaneously. As illustrated in the simulation of a gas turbine engine fault detection, a significant improvement of disturbance attenuation is achieved compared with the existing methods. The main contribution of this paper is to combine the frequency estimation and eigenvalue optimization in eigenvalue assignment for RFDO design. The benefits of this method are two fold: the reduction of the computation costs in RFDO design, and the avoidance of the local optimal solution due to fixed eigenvalues.

II. Problem Formulation

Consider a disturbance-corrupted system with faults in the discrete state space form:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + B_f f(k) + B_d d(k) \\ y(k) = Cx(k) + Du(k) + D_f f(k) + D_d d(k) \end{cases} \qquad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$. $f(k) \in \mathbb{R}^f$ is a general fault vector, B_f, D_f are known as *fault distribution matrices*, and B_d, D_d are termed *disturbance distribution matrices*. $d(k) \in \mathbb{R}^d$ is a general disturbance vector due to exogenous signals, linearization or parameter uncertainties. For instance, the disturbance caused by model uncertainties can be presented as:

$$d(k) = \begin{pmatrix} \Delta Ax(k) + \Delta Bu(k) \\ \Delta Cx(k) + \Delta Du(k) \end{pmatrix} \qquad (2)$$

In this paper, $d(k)$ is assumed as a quasi-stationary process with both deterministic and stochastic components:

$$d(k) = s(k) + h(k) * n(k) \qquad (3)$$

where $s(k)$ is a band-limited deterministic disturbance vector, $n(k)$ a white noise, $h(k)$ the impulse response of a band-pass filter having the similar band as $s(k)$, and $*$ denotes the convolution product. Thus, $h(k) * n(k)$ is a band-limited stationary stochastic signal (colored noise). It can be proved that $d(k)$ is quasi-stationary and band-limited. Without loss of generality, it is assumed that the pair $\{A, C\}$ is observable.

For system (1), the robust fault detection observer under consideration can be constructed by

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + Bu(k) + Kr(k) \\ \hat{y}(k) = C\hat{x}(k) + Du(k) \\ r(k) = y(k) - \hat{y}(k) \end{cases} \qquad (4)$$

where $r \in \mathbb{R}^p$ is the so-called residual which is evaluated to determine the system is faulty or not.

Define the state estimation error $e(k) = x(k) - \hat{x}(k)$, the estimation error and residual dynamics are governed by

$$\left\{ \begin{array}{l} e(k+1) = (A-KC)e(k) + (B_f - KD_f)f(k) \\ \qquad\qquad +(B_d - KD_d)d(k) \\ r(k) = Ce(k) + D_f f(k) + D_d d(k) \end{array} \right. \tag{5}$$

The z-transformation of (5) gives the Transfer Function Matrices (TFMs) relating $r(z)$ to $f(z)$, $d(z)$:

$$r(z) = G_f(z)f(z) + G_d(z)d(z) \tag{6}$$

where

$$\left\{ \begin{array}{l} G_f(z) = C(zI - A + KC)^{-1}(B_f - KD_f) + D_f \\ G_d(z) = C(zI - A + KC)^{-1}(B_d - KD_d) + D_d \end{array} \right. \tag{7}$$

It can be seen from (6) that, due to the existences of disturbances, the residual $r(z)$ is not zero, even if no fault occurs. The effect of disturbances works as a source of false and missed alarms. In order to avoid false alarms, the concept of RFDO was proposed in a lot of literatures aiming to reduce the effects of disturbances and to enhance the effects of faults.

The RFDO problem to be solved in this paper turns into a constrained optimization problem:

RFDO Design Given a system (1) subject to constant/slow faults and unknown disturbances $d(k)$ limited on some frequency ω_d, find, if possible, a real coefficient feedback gain matrix $K \in \mathbb{R}^{n\times p}$, such that the following two criteria are satisfied:

- **Stability Criterion:** The eigenvalues of $A - KC$ in equation (5) lie within the unit circle in the z-plane.
- **Robustness/Sensitivity Criterion:** $\| G_d(z) \|$ should be minimized and $\| G_f(z) \|$ should be maximized, where $\| \cdot \|$ denotes some kind of TFMs norm.

III. Disturbance Attenuation Design

Motivated by the work of Gao and Wang [15], the left eigenvector assignment is employed in this paper. The objectives are to define a more appropriate performance index by taking into account the frequency properties of disturbances and then minimize it by selecting an optimal matrix $K \in \mathbb{R}^{n\times p}$.

In the followings, the gain matrix K in (4) and the TFMs (7) are first parameterized by eigenvalues $\{\lambda_i\}$ and free parameters $\{q_i\}$. Then the evaluation of robustness/sensitivity index $\frac{\|G_d(z)\|}{\|G_f(z)\|}$ is discussed. A disturbance frequency estimation method is proposed to reduce the computation complexity and improve the performance index. Finally, the optimization procedure is slightly modified so that not only the free parameters, but also the eigenvalues are optimized. The following assumption is used throughout:

Assumption A1 The poles λ_i $(i = 1, 2, \ldots, n)$ of the closed loop observer (4) are distinct from those of the open loop plant system (1).

A. Eigenstructure Parameterization

Derived from [1], [10], [13], [16], the parametric expression of the gain matrix K can be expressed as:

Lemma 1: Let $\{A, C\}$ be observable, then, for any group of scalars λ_i, $i = 1, 2, \ldots, n$ under assumption A1, the gain matrix K can always be parameterized as:

$$K = L^{-1}Q \tag{8}$$

where $L \in \mathbb{R}^{n\times n}$ is composed of the left eigenrows l_i of $A - KC$, corresponding to the eigenvalue λ_i respectively,

$$L = \left[\begin{array}{c} l_1^T \\ \vdots \\ l_n^T \end{array} \right] = \left[\begin{array}{c} q_1^T C(A - \lambda_1 I)^{-1} \\ \vdots \\ q_n^T C(A - \lambda_n I)^{-1} \end{array} \right] \tag{9}$$

and $Q \in \mathbb{R}^{n\times p}$ is composed of the free parameter vectors q_i

$$Q^T = [q_1^T \; q_2^T \; \ldots \; q_n^T] \tag{10}$$

Lemma 1 gives an explicit, parametric expression of K, with the eigenvalues λ_i and the vectors q_i as the free parameters. Except for the assumption A1, there are two more constraints in Lemma 1:

Constraint C1 The set $\{\lambda_i\}$ and the set $\{q_i\}$ of free parameter vectors must be self-conjugated such that the resulting feedback matrix K is a real coefficient matrix.

Constraint C2 In order that the vectors l_i are the closed-loop eigenvectors, λ_i and q_i must be chosen in such a way that l_i yields linearly independent vectors. Then left eigenvector matrix K is non-singular and the inverse K^{-1} exists.

As a result, the discrete TFMs $G_f(z)$, $G_d(z)$ can also be expressed in a parametric form.

Lemma 2: Under the assumption A1 and constraints C1, C2, the discrete transfer function matrices (7) can be expressed in terms of the eigenstructure, respectively, as

$$\begin{array}{l} G_d(z) = D_d + C\ [R\Psi(z)L]\ (B_d - L^{-1}QD_d) \\ G_f(z) = D_f + C\ [R\Psi(z)L]\ (B_f - L^{-1}QD_f) \end{array} \tag{11}$$

where

$$R = L^{-1} = (r_1, r_2, \ldots, r_n) \tag{12}$$

and

$$\Psi(z) = \left[\begin{array}{cccc} \frac{1}{z-\lambda_1} & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \frac{1}{z-\lambda_n} \end{array} \right] \tag{13}$$

Proof. Similar to the Lemma 10.2 in [10], by replacing the s-transformation with the z-transformation, the inverse of any discrete TFM $(zI - A + KC)^{-1}$ can be expanded as

$$(zI - A + KC)^{-1} \quad = \frac{r_1 l_1^T}{z-\lambda_1} + \frac{r_2 l_2^T}{z-\lambda_2} + \ldots + \frac{r_n l_n^T}{z-\lambda_n} \tag{14}$$

Substituting (14) into (7) gives the parametric expressiones of $G_d(z)$ and $G_f(z)$. Rewriting it into matrix form, (11) is thus given. Proved.

It is worth to note that all the vectors l_i, r_i (matrices L, R) depend on the choice of λ_i and q_i $(i = 1, 2, \ldots, n)$. λ_i can be arbitrarily chosen from the fields $\mathbb{C}$ of real (or complex) numbers, and q_i from the real vector spaces $\mathbb{R}^p$, respectively. The freedom allows to use a lot of existing optimization algorithms to find the optimal gain matrix K. Before optimization, an issue needed to be addressed is the definition and evaluation of the robustness/sensitivity criteria and related TFM-norms.

B. Performance Index Evaluation

One of widely accepted robustness/sensitivity criteria is the H_∞-norm based index: $J_{\infty/\infty} = \frac{\|G_d(z)\|_\infty}{\|G_f(z)\|_\infty}$, where H_∞-norm is used to measure the largest singular value of a TFM over the whole frequency range. Minimizing $J_{\infty/\infty}$ is to find a matrix K so that the fault detection is optimal at the worst case. However the H_∞ optimal observer may be too conservative, because it only minimizes the peak value to give the basic guarantee of system performance. A further drawback is the computation complexity. H_∞-norms are calculated by gridding and integrating over the whole frequency range $0 \leq$

$|\omega| \leq \pi$, which introduce more computational burden and then is not suitable for on-board fault detection of GTEs. In order to avoid these drawbacks, a modified performance index is proposed as follows.

1) Robustness Index: Based on the observation that most disturbances in GTE systems are frequency band limited, a modified robustness index is proposed here through evaluating the TFM-norm at the disturbance frequency point $z = e^{jw_d}$, in stead of the whole frequency range:

$$\min_{Q,\Lambda} \|G_d(z)\|_{z=e^{jw_d}} \tag{15}$$

where the disturbance is assumed mainly concentrated at some frequency w_d, $0 \leq |w_d| \leq \pi$.

Similar to H_∞ theory, here the disturbance and w_d is still unknown. The following theorem is used to solve the problem of disturbance frequency estimation:

Theorem For a discrete system (1), under the assumption that the disturbances is frequency band-limited, if the observer (5) is stable, at steady state, the main spectrum set of residual, Ω_r, is limited to a portion of Ω_d, the spectrum set of disturbance. That is $\Omega_r \subseteq \Omega_d$

The result of this theorem is standard when applied to univariate system and the extension to multivariable system appears to be new. It can be simply interpreted as: for an discrete-time observer, the disturbance frequency w_d of $d(k)$ has not changes and then can be identified from residual $r(k)$.

It follows that, if the spectrum of residual mainly lies at frequency w_r, then (15) can be computed as

$$\min_{Q,\Lambda} J_1 = \|G_d(z)\|_{z=e^{jw_r}} \tag{16}$$

2) Sensitivity Index: Not like a random noise, a fault signal is associated with some pattern and, from the viewpoint of frequency domain, its distribution is not uniform over the whole frequency range. For instance, an incipient fault comprises mainly low frequency components. For abrupt faults, high frequency contents only exist at the time instant when faults start, and it is almost constant (zero frequency) content thereafter. For detecting these common faults mainly on low frequency, the steady state gain is the most important factor and Chen etc. [1] proposed strong fault detectability condition: $\|G_f(s)\|_{s=0} \neq 0$ in continuous time domain. In this paper, it is proposed that the $\|G_f(z)\|_{z=1}$ index should be maximized for increasing the fault significance, which gives

$$\max_{Q,\Lambda} J_2 = \|G_f(z)\|_{z=1} \tag{17}$$

Generally, for a fault with main frequency components at frequency w_f and w_f is known a priori, the sensitivity index can be defined as

$$\max_{Q,\Lambda} J_2 = \|G_f(z)\|_{z=e^{jw_f}} \tag{18}$$

Combining the robustness index (16) and sensitivity index (18) leads to the performance index as:

$$\min_{Q,\Lambda} J = \frac{J_1}{J_2} = \frac{\|G_d(z)\|_{z=e^{jw_r}}}{\rho + \|G_f(z)\|_{z=e^{jw_f}}} \tag{19}$$

where ρ is a small positive real number. The aim is to avoid division by zero when $\|G_f(e^{jw_f})\|$ is zero in some cases.

Remark 1: Since the variables z in (19) are given specific values w_r or w_f, respectively, the computation of the TMF-norm is converted into a numerical matrix norm calculation. Compared to the H_∞ TMF-norm, which requires griding, computing and finding the largest singular value over the whole frequency $[0, \pi]$, performance index (19) only involves the computation of two real matrices. The associated computation is very low. This benefit is paid by the spectrum analysis which can be effectively carried out by using FFT.

Remark 2: It is worthy note that the spectrum set of residuals is just a part of the spectrum set of disturbances. For some disturbance $e^{j\omega_0 k}$, if the magnitude $\|G_d(z)\|_{z=e^{j\omega_0}} = 0$, then ω_0 does not appear in Ω_r. It means the residual $r(k)$ is not affected by the disturbance $d(k)$. Hence, it is not necessary to attenuate such a disturbance.

Remark 3: As the frequency information is incorporated into the new index (19), the resulting observer is optimal for attenuating such a certain disturbance. In most applications, such an observer has a better disturbance attenuation performance.

Remark 4: Compared to FFT-based fault detection methods (e.g., MCAS in electrical motor condition monitoring), the advantage is again the relatively low computation cost. In common FFT-based fault detections, the FFT has to be repeated when new data arriving. In our method, the FFT only performances once for disturbance estimation at the observer design step. At the fault detection step, the observer parameters keep unchanged and faults are detected in time domain by comparing the observer outputs with actual outputs. Hence, a lot of FFTs are avoided and the computation burden is relatively small.

C. Optimization of free parameters and eigenvalues

It can be seen from (11), the performance function J (19) is not only a function of the free parameters $\{q_i\}$, but also a function of the eigenvalues $\{\lambda_i\}$. The values of $\{\lambda_i\}$ not only determine the stability, but also affect the performance index to a great extent. In most papers, however, only $\{q_i\}$ are optimized and the eigenvalues are given a prior. Those optimizations are more likely local optimal [11].

An alternative way to improve the disturbance rejection performance is to optimize both poles Λ and free parameters Q simultaneously. In this paper, no exact positions of poles are pre-specified, whereas the regions where poles should lie in are specified according to the stability and response speed requirement.

Based on the discussion above, the solution to the RFDO can now be stated as the follows:

If assumption A1 and constraints C1, C2 are satisfied, and the main frequency contents of residuals $r(k)$ can be estimated at w_r, then minimizing the following performance index

$$J(Q,\Lambda) = \frac{\left\|D_d + CR\Psi(z)L(B_d - L^{-1}QD_d)\right\|_{z=e^{jw_r}}}{\rho + \left\|D_f + CR\Psi(z)L(B_f - L^{-1}QD_f)\right\|_{z=e^{jw_f}}} \tag{20}$$

gives the optimal gain matrix $K = L^{-1}Q$ such that the disturbance $d(k)$ is attenuated and the sensitiveness to faults is enhanced to the greatest extent.

IV. Application and Results

To illustrate the proposed RFDO design approach, this section presents results of an application to the detection of actuator faults of a gas turbine engine. Real engine fuel flow data gathered from normal engine closed-loop operation at the engine test-bed are used [14].

In aero engines, the main characteristics are the dynamics between the fuel flow and shaft speeds. The control system is usually organized as a dual-lane system with two sets of parallel sensors and controllers [14]. A general scheme is presented in Fig. 1, where the input is the flow rate W_f, the outputs are the low pressure shaft speed N_{lp} and the high pressure shaft speed N_{hp}. The vectors Y_{s1}, Y_{s2} denote the measurements of $[N_{lp}, N_{hp}]^T$ by the two set of sensors respectively. Y_{s3} denotes the model prediction value of $[\hat{N}_{lp}, \hat{N}_{hp}]^T$.

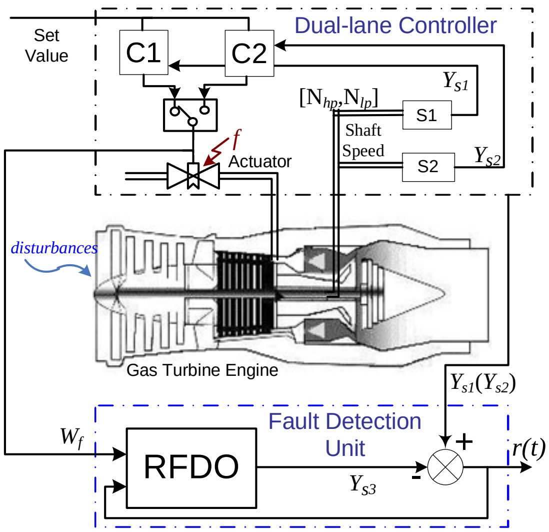

Fig. 1. Dual-lane control of gas turbine engines. Controller C1 and sensor S1 compose the primary lane. Controller C2 and sensor S2 compose a spare lane and is waiting in "hot back-up." When the primary lane fails, the spare lane comes online immediately. The model works as the third virtual lane.

In this application, the detection of actuator faults is the focus and both sensor sets are assumed fault-free. Generally, there are two categories of actuator faults: abrupt faults and incipient faults. An abrupt fault is that a machine breaks down without any warning of impending failure(e.g., blocked filters/valves, sudden pipe leakage) and an incipient fault is a gradual process with a deteriorating fault condition (e.g. drift failure, deterioration of actuator, blade containments). Particularly, the earlier detection of incipient faults has a lot of benefits for reliable operation and reducing maintenance costs.

A reduced order model of the GTE is identified by using the method in [17] and expressed in the state space form

$$\left\{\begin{aligned} \begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} &= \begin{bmatrix} 0.9769 & 0.0038 \\ 0.0936 & 0.9225 \end{bmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} \\ &\quad + \begin{bmatrix} 2.1521 \\ 8.8186 \end{bmatrix} W_f(k) \\ \begin{pmatrix} N_{hp}(k) \\ N_{lp}(k) \end{pmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} \end{aligned}\right. \tag{21}$$

with the sampling interval $T_s = 0.025$ second. It is easy to verify that the system (21) is observable and the open-loop poles are $[0.9828, 0.9166]$. The disturbance model is assumed as

$$\begin{aligned} B_d &= \begin{bmatrix} 0.1510 & 0.0406 \\ 0.0500 & 0.0528 \end{bmatrix}, & D_d &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\ B_f = B &= \begin{bmatrix} 2.1521 \\ 8.8186 \end{bmatrix}, & D_f &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned} \tag{22}$$

where the fault matrix $B_f = B$ for actuator fault. The disturbance injected to the system is simulated by

$$\left\{\begin{aligned} d_1(k) &= s_1(k) + h(k) * n_1(k) \\ d_2(k) &= s_2(k) + h(k) * n_2(k) \end{aligned}\right. \tag{23}$$

where $[n_1(k), n_2(k)]$ are white noises with covariance matrix $[0.4\ 0;\ \ 0\ 0.04]$ and zero mean values. $h(k)$ is a filter with pass band of $[1.5, \pi]$. The deterministic signals are

$$\left\{\begin{aligned} s_1(k) &= \sin(2k) + \cos(2.1k + \pi/4) + \\ &\qquad 0.5\cos(2.3k) + 0.5\sin(2.2k - \pi/4) \\ s_2(k) &= 0.5\sin(2.1k) + 0.25\sin(1.5k + \pi/4) \end{aligned}\right. \tag{24}$$

The injected disturbance is shown in Fig. 2 and its 128-point FFT based spectrum is shown in Fig. 4(a).

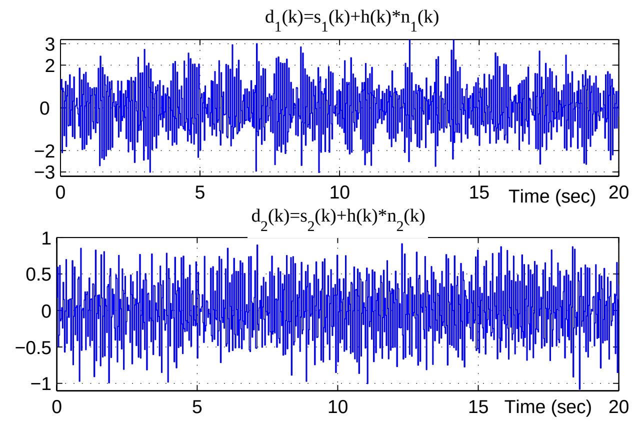

Fig. 2. Disturbances injected into the system

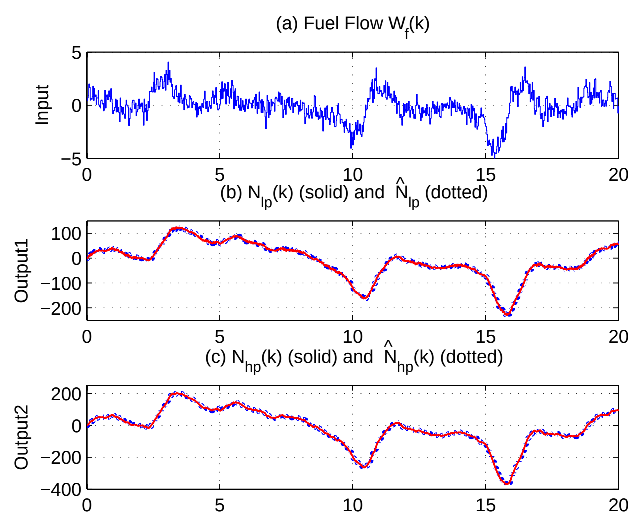

Fig. 3. Experiment: the input W_f, the outputs of the shaft speeds ($[N_{lp}, N_{hp}]^T$) and their estimations $[\hat{N}_{lp}, \hat{N}_{hp}]^T$ predicted by the observer K_0. Since the model is identified at a steady operating point, the data used here has been subtracted from the operating point equilibrium.

In order to estimate the disturbance frequency, a gain matrix K_0 is first constructed via *place(A',C',[0.5 0.5])'*. The inputs, outputs of observer K_0 are shown in Fig.3. A 128-point FFT is employed to calculate the spectrum of $r(k)$, as shown in Fig. 4(b). Compared to the spectrum of disturbances (Fig. 4(a)), $r(k)$ is a band-limited quasi-stationary signal and has main frequency components around $w_d = 2.1$.

The desired poles region are set as $|\lambda_i| < 0.75$, which is a round centering at the origin. $[-0.5\ 0.5]$ are set as the initial values of λ_i. The initial value of Q are

$$Q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.1 \\ 1.0 & 0.0 \end{pmatrix} \tag{25}$$

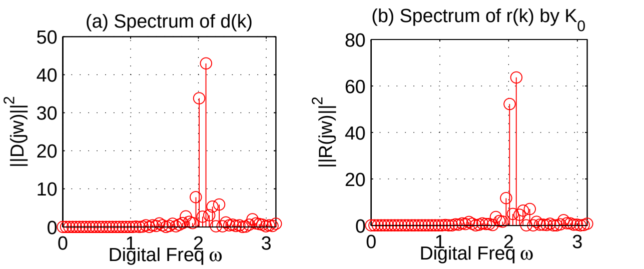

Fig. 4. (a) Disturbance spectrum and (b) its frequency estimation through 128-point FFT-based residuals spectrum analysis

It is easy to verify that the assumption A1 and constraints C1, C2 are satisfied. The value of ρ in (19) is set as 0.1.

Using *fmincon* provided by MATLAB Optimization Toolbox gives the optimal gain matrix K

$$K_{opt} = \begin{pmatrix} 3.3769 & -11.0584 \\ 0.9516 & -2.8394 \end{pmatrix} \tag{26}$$

with optimal poles [0.6519, 0.7100] and $J = 0.0005583$. For comparison, K_{place} (by *place* command provided by MATLAB) and K_{inf} (by H_∞ method) are designed,

$$K_{place} = \begin{pmatrix} 0.3250 & 0.0038 \\ 0.0936 & 0.2125 \end{pmatrix}, K_{inf} = \begin{pmatrix} 0.2402 & 0.0549 \\ 0.0493 & 0.2973 \end{pmatrix} \tag{27}$$

where K_{place}, K_{inf} have identical eigenvalues as K_{opt}.

A. Residuals without Faults

Fig. 5 shows the residuals $\|r(k)\|_{opt}$, $\|r(k)\|_{place}$, $\|r(k)\|_{inf}$ and their spectra, respectively. The disturbance attenuation of K_{opt} is more apparent compared to that of K_{place}, K_{inf}. In the time domain, it can been seen that the maximum magnitude of $\|r(k)\|_{opt}$ is below than 3 in the steady state, however that of $\|r(k)\|_{place}$ and $\|r(k)\|_{inf}$ are nearly 4. In the frequency domain, the disturbance attenuation of K_{opt} is obviously better. Particularly, the residual spectrum magnitude of observer K_{opt} at $\omega_d = 2.1$ is attenuated to 20. The performance of disturbance attenuation is expressed by the ratio of the power of $d(k)$ and $r(k)$ in decibel (dB), as shown

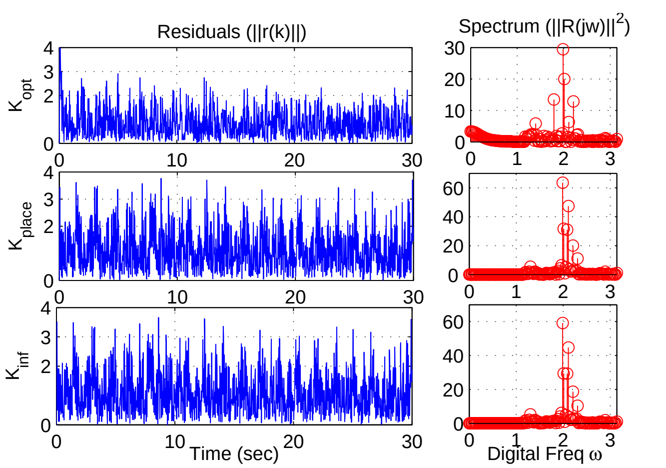

Fig. 5. Residuals $\|r(k)\|_{opt}$, $\|r(k)\|_{place}$ and $\|r(k)\|_{inf}$ corresponding to K_{opt} K_{place}, K_{inf} respectively in the fault free case (left column) and their associated spectra (right column)

$$\textbf{dB} = 10\log_{10}\frac{\|R(jw)\|^2}{\|D(jw)\|^2} \tag{28}$$

At frequency 2.1, the disturbance is attenuated -8 **dB** by K_{opt}. Whereas it is 0 **dB** in K_{place} and K_{inf}.

The benefit of the smaller residual amplitude of $\|r(k)\|_{opt}$ is that K_{opt} is able to detect a smaller fault and to avoid false alarms.

B. Detection of actuator faults

Although many actuator faults lead to an abrupt changes, in practice, actuator faults can also be caused by the components degradation and behave as slow changes. Such faults are extremely difficult to be detected immediately from a simple visual inspection of the output signals. To simulate the incipient fault of the fuel pump gain drift of 0.002 unit per second, the fault function $f(t)$ is represented as

$$f(t) = \begin{cases} 0 & (t \leqslant 10.05) \\ 0.002(t-10.05) & (10.05 < t < 20.05) \\ 0.008 & (t \geqslant 20.05) \end{cases} \tag{29}$$

This is a typical saturated actuator fault caused by component degradation. Fig. 6 shows the norms of the residual vectors. The observers K_{place}, K_{inf} fail to detect such a fault, as there is no obvious changes in their residuals. However, $\|r(k)\|_{opt}$ shows an increase soon after the fault happening, and then follows the fault. From the view point of fault detection delay, it is less than 5 second after the fault happening when K_{opt} gives fault indication and no false alarm thereafter. However, even 20 seconds later, both K_{place}, K_{inf} fail to detect the fault. This verifies that K_{opt} is able to detect an incipient fault earlier and more distinctly.

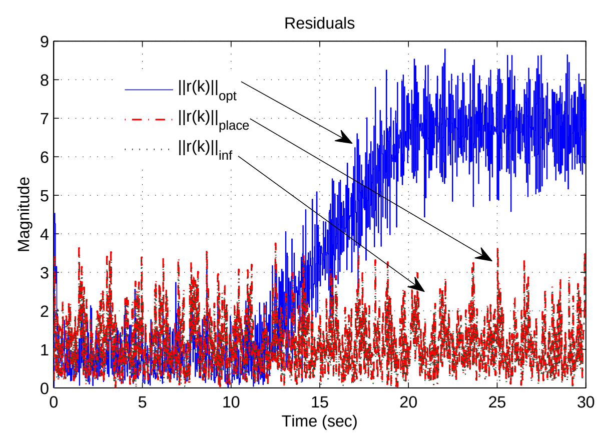

Fig. 6. Residuals of K_{opt}, K_{place} and K_{inf} in the case of a saturated incipient actuator fault.

V. Conclusion

In this paper, a robust fault detection observer has been designed through FFT-based disturbance frequency estimation and left eigenvector-based eigenvalue assignment. Compared to the existing methods, the benefits of the proposed approach are two fold: First, the computation costs is reduced, which makes this approach more suitable for on-board condition monitoring of GTEs. By estimating and integrating disturbance frequency information, $\|G_d(z)\|$ and $\|G_f(z)\|$ are evaluated at frequency $z = e^{jw_r}$, $z = e^{jw_f}$, respectively. Thus, a numerical matrix norm is calculated, rather than the time-consuming H_∞-norm of transfer function matrix. Second, by optimizing free parameters Q and eigenvalues $\{\lambda_i\}$ simultaneously, this technique is more likely to avoid a local minimal and able to give better disturbance attenuation.

In the application to the actuator faults detection of a GTE, the improvement on disturbance attenuation and fault detection has been demonstrated. Although this is designed for attenuating external disturbances, the principles used here are applicable to the problem of model uncertainty. Further study is needed to solve it. Finally, when applying this method to real embedded systems, some issues (e.g., calculation accuracy, computation complexity) needs further study.

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